Tropical Geometry and Mirror Symmetry Seminar

Semester: Fall Semester 2010
Date: Th. 5-6 pm.
Room: 891 Evans Hall, UC Berkeley.
Course control #: 54581
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Schedule

We will follow Gross’ preliminary version of a book [Kansas], complemented by further reading. Additionally to the given reference here, please also be aware of those given by the end of each chapter in [Kansas].

Aug. 26th: Fix seminar schedule, speakers and give first lecture (overview).

Sept. 2nd: Tropical Geometry I, Basics of Toric Geometry ([Kansas] Sections 1.1–1.3, 3.1): Tropical semiring, tropical “zero loci”, (baby version of) discrete Legendre transform, fans (1.2), skip Example 1.6, go as far as Prop. 1.15 (including it), also cover ([Kansas] Section 3.1)!

Sept. 9th: Tropical Geometry II ([Kansas] Sections 1.3-1.4). Continue after Prop. 1.15, moduli of tropical curves, counting multiplicities, tropical Bezout (Ex.1.6), tropical manifolds, 2-3dim. singularities.

Sept. 16th: From Batyrev-Borisov-polytopes to tropical manifolds. This talk is devoted on how to get a tropical manifold from a reflexive polytope or a reflexive polytope with nef partition (Batyrev, Batyrev-Borisov construction). One source for this is [Gross98] which is summarized in [GrossProc]. The speaker may also fill in as much of [HZI, HZII, HZIII] as he feels comfortable with.

Sept. 23rd: Tropical Geometry III ([Kansas], Sections 1.5-1.6, 4.1). Discrete Legendre transform (refer to previous talk by making the boundary of a reflexive polytope be an example), tropical curves on surfaces and most importantly: Overview of Mikhakin’s curve counting formula and proof strategy.

Sept. 30th: Introduction to Log Geometry ([Kansas], Chapter 3). Try to give a feeling for how log geometry works without being too technical, e.g. say why $xy = 0$ is log smooth even though the underlying space is very singular. A listener should have an idea about what log smooth maps are after the talk. A glance at ([Kansas], Chapter 4.2) to see what is needed later on will help.
Oct. 7th: Introduction to stable maps and Gromov-Witten invariants. ([Kansas] Section 2.1.1). Try to find and compute nice examples to relate Gromov Witten invariants to classical algebraic geometry: Count rational curves in \(\mathbb{P}^2\) (Example 2.6), and possibly present the list on p.2 in Aaron Bertram’s introduction http://www.math.utah.edu/~bertram/courses/nice/ps/intro.ps. If it hasn’t appeared in the seminar yet, also draw some tropical examples, e.g. how many points determine a tropical line, conic or cubic in the plane.

Oct. 14th: Mikhalkin’s curve counting formula I ([Kansas], Section 4.2-4.3).

Oct. 21th: Mikhalkin’s curve counting formula II ([Kansas] Section 4.4–4.6).

Oct. 28th.: The A-model ([Kansas] 2.1.2-2.1.4). Quantum cohomology, WDVV equation, Frobenius manifolds, quantum differential equation

Nov. 4th.: The B-model ([Kansas] 2.1.5-2.1.7; 2.2.3-2.2.5). Semi-infinite variation of Hodge structure, moving subspace, relation to Frobenius manifolds, B-model data for \(\mathbb{P}^n\).

Nov. 11th.: Summary talk: What have we learned so far? Recalling examples, retelling the story.

Nov. 18th.: The Tropical B-model I ([Kansas] 5.1-5.2). Givental’s mirror in 2dim, tropical disks, Maslov index, tropical descendent invariants.

Nov. 25th.: Thanksgiving (no seminar)

Dec. 2nd: Guest speaker: Eric Katz, topic TBD

Dec. 11th: The Tropical B-model II ([Kansas] 5.3-5.5). Maslov index 0 and 2 disks, scattering diagrams, broken lines, wall-crossing theorem, Evaluation of the period integrals

References


[Gross01] Gross, M., Topological mirror symmetry Inventiones Mathematicae 144, Number 1 / April 2001


